

Aufgaben zum Lösen von Bruchgleichungen

1.0 Bestimmen Sie die Definitions- und die Lösungsmenge.

$$1.1 \quad \frac{1}{x} - \frac{x}{1+x} = -1$$

$$1.2 \quad \frac{0,4}{3x} + \left(\frac{0,1}{x} - \frac{1,5}{4} \right) \cdot 2 = 0$$

$$1.3 \quad \frac{x^2 - 4}{3x} - \frac{2-x}{3} = \frac{2x}{3} - \frac{4}{x}$$

$$1.4 \quad \frac{1}{x-2} = \frac{2}{x^2 - 2x}$$

$$1.5 \quad \frac{x+8}{3x+3} + \frac{x+2}{2x+2} = 1$$

$$1.6 \quad \frac{2x-30}{x+3} - 2 = 6 \cdot \frac{x+3}{x^2-9}$$

$$1.7 \quad \frac{x^2 + 3x + 4}{x^2 + 3x + 2} = 1$$

$$1.8 \quad \frac{x+1}{x-1} - \frac{3}{x-1} = 1$$

2.0 Bestimmen Sie die Definitionsmenge und lösen Sie folgende Gleichungen durch Überkreuzmultiplikation.

$$2.1 \quad \frac{x}{x+2} = \frac{2x}{2x+9}$$

$$2.2 \quad \frac{x+6}{x} = \frac{x+4}{x+1}$$

$$2.3 \quad \frac{x}{x-7} = \frac{x+5}{x-3}$$

$$2.4 \quad \frac{3x+2}{3x-1} = \frac{6x}{6x-1}$$

$$2.5 \quad \frac{2}{x-1} = x$$

$$2.6 \quad \frac{2x+1}{3x-4} = \frac{x+6}{4x+1}$$

Lösungen

1.1

$$D = \mathbb{R} \setminus \{-1; 0\}$$

$$\frac{1+x-x^2}{x \cdot (1+x)} = -1 \Rightarrow 1+x-x^2 = -x-x^2 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2} \Rightarrow IL = \left\{-\frac{1}{2}\right\}$$

1.2

$$D = \mathbb{R} \setminus \{0\}$$

$$\frac{0,4}{3x} + \frac{0,2}{x} - \frac{3}{4} = 0 \Rightarrow \frac{0,4+3 \cdot 0,2}{3x} = \frac{3}{4} \Rightarrow \frac{1}{3x} = \frac{3}{4} \Rightarrow 4 = 9x \Rightarrow x = \frac{4}{9} \Rightarrow IL = \left\{\frac{4}{9}\right\}$$

1.3

$$D = \mathbb{R} \setminus \{0\}$$

$$\frac{x^2-4}{3x} + \frac{4}{x} = \frac{2x}{3} + \frac{2-x}{3} \Rightarrow \frac{x^2-4+12}{3x} = \frac{x+2}{3} \Rightarrow \frac{x^2+8}{3x} = \frac{x+2}{3} \\ \Rightarrow 3x^2+24=3x^2+6x \Rightarrow 6x=24 \Rightarrow x=4 \Rightarrow IL = \{4\}$$

1.4

$$D = \mathbb{R} \setminus \{0; 2\}$$

$$x^2-2x=2x-4 \Rightarrow x^2-4x+4=0 \Rightarrow x=2 \Rightarrow IL = \{ \}$$

1.5

$$D = \mathbb{R} \setminus \{-1\}$$

$$\frac{x+8}{3x+3} = 1 - \frac{x+2}{2x+2} \Rightarrow \frac{x+8}{3x+3} = \frac{2x+2-x-2}{2x+2} \Rightarrow \frac{x+8}{3x+3} = \frac{x}{2x+2} \\ \Rightarrow 3x^2+3x=2x^2+18x+16 \Rightarrow x^2-15x-16=0 \Rightarrow x_1=16 \quad x_2=-1 \Rightarrow IL = \{16\}$$

1.6

$$D = \mathbb{R} \setminus \{-3; 3\}$$

$$\frac{2x-30-2x-6}{x+3} = \frac{6x+18}{x^2-9} \Rightarrow \frac{-36}{x+3} = \frac{6x+18}{x^2-9} \Rightarrow -36x^2+324=6x^2+36x+54 \\ \Rightarrow -42x^2-36x+270=0 \Rightarrow x_1=-3 \quad x_2=\frac{15}{7} \Rightarrow IL = \left\{\frac{15}{7}\right\}$$

1.7

$$D = \mathbb{R} \setminus \{-2; -1\} \quad x^2+3x+2 = (x+1) \cdot (x+2)$$

$$x^2+3x+4 = x^2+3x+2 \Rightarrow 4=2 \text{ (f)} \Rightarrow IL = \{ \}$$

1.8

$$D = \mathbb{R} \setminus \{1\}$$

$$\frac{x+1-3}{x-1} = 1 \Rightarrow x-2 = x-1 \Rightarrow -2 = -1 \text{ (f)} \Rightarrow IL = \{ \}$$

2.1

$$D = \mathbb{R} \setminus \{-4, 5; -2\}$$

$$2x^2 + 9x = 2x^2 + 4x \Rightarrow 5x = 0 \Rightarrow x = 0 \Rightarrow IL = \{0\}$$

2.2

$$D = \mathbb{R} \setminus \{-1; 0\}$$

$$x^2 + 7x + 6 = x^2 + 4x \Rightarrow 3x = -6 \Rightarrow x = -2 \Rightarrow IL = \{-2\}$$

2.3

$$D = \mathbb{R} \setminus \{3; 7\}$$

$$x^2 - 3x = x^2 - 2x - 35 \Rightarrow -x = -35 \Rightarrow x = 35 \Rightarrow IL = \{35\}$$

2.4

$$D = \mathbb{R} \setminus \left\{ \frac{1}{6}; \frac{1}{3} \right\}$$

$$18x^2 + 9x - 2 = 18x^2 - 6x \Rightarrow 15x = 2 \Rightarrow x = \frac{2}{15} \Rightarrow IL = \left\{ \frac{2}{15} \right\}$$

2.5

$$D = \mathbb{R} \setminus \{1\}$$

$$2 = x^2 - x \Rightarrow x^2 - x - 2 = 0 \Rightarrow x_1 = -1 \quad x_2 = 2 \Rightarrow IL = \{-1; 2\}$$

2.6

$$D = \mathbb{R} \setminus \left\{ -\frac{1}{4}; \frac{4}{3} \right\}$$

$$8x^2 + 6x + 1 = 3x^2 + 14x - 24 \Rightarrow 5x^2 - 8x + 25 = 0 \Rightarrow IL = \{ \}$$